Preparation For AP Calculus

Assignments

p 9 1-30, 43-50
p 10 51, 52, 54
p 17 1-12, 21-34
p 18 43-48, 55, 56
p 28 1-16, 27, 28
p 29 40-44, 47-50

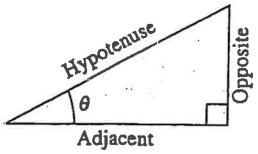
Also memorize the unit circle and all checked identities and formulas

Memorize all check items.

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

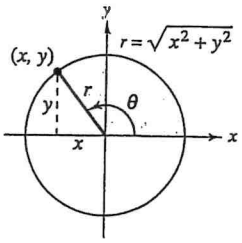


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

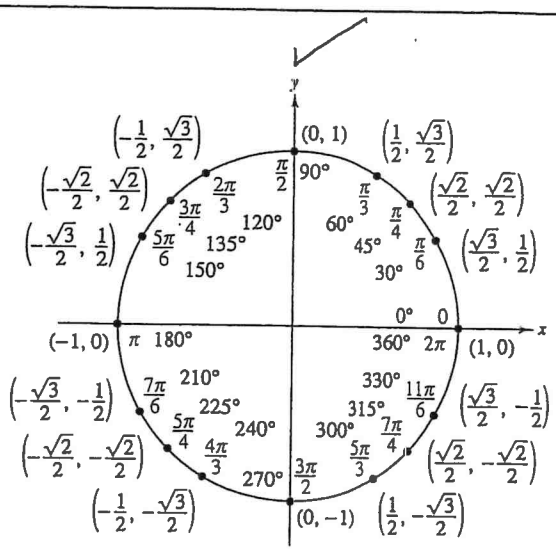
Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

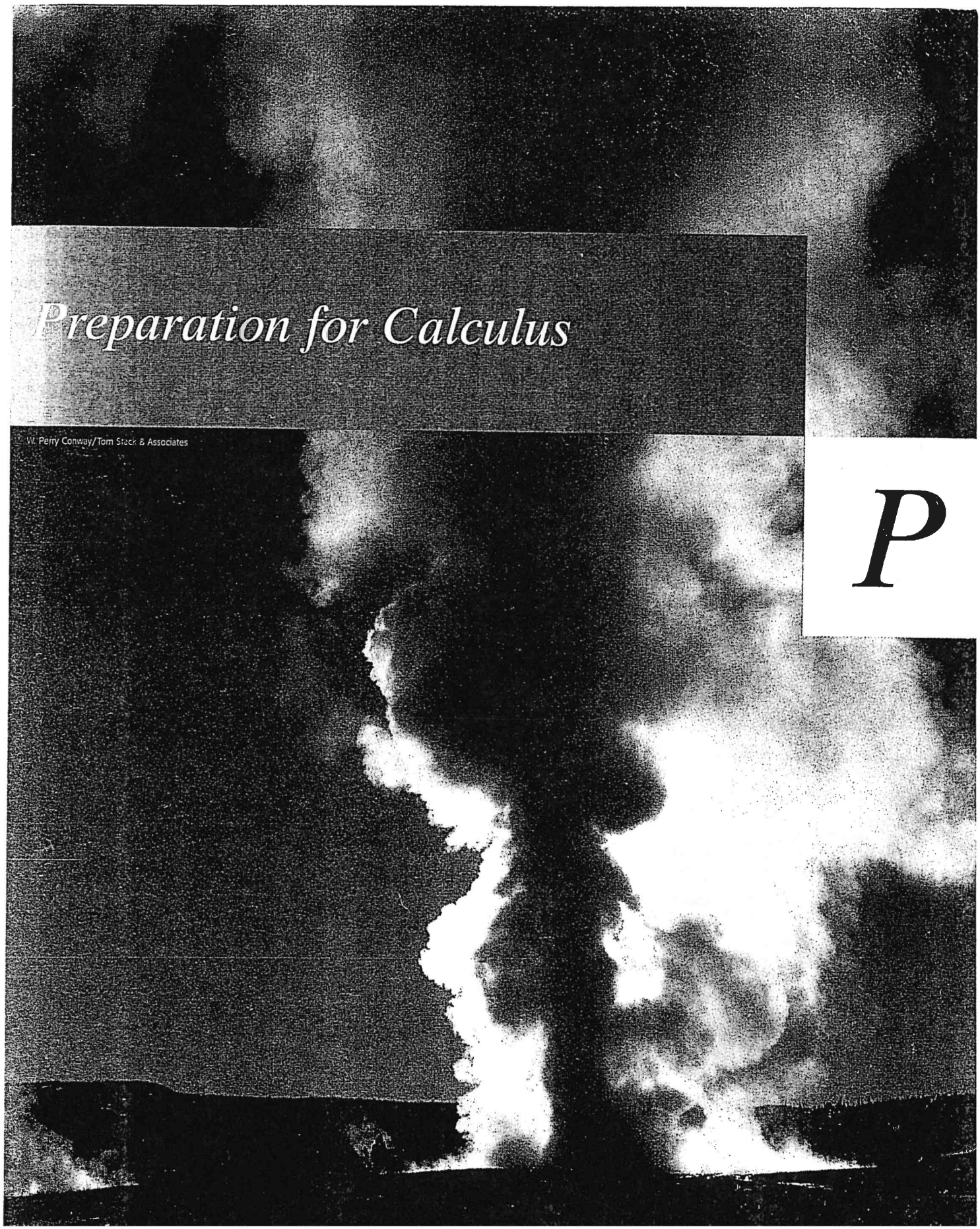
Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$



Preparation for Calculus

W. Perry Conway/Tom Stack & Associates

P

SECTION P.1 Graphs and Models

The Graph of an Equation • Intercepts of a Graph • Symmetry of a Graph • Points of Intersection • Mathematical Models


Archive Photos

RENÉ DESCARTES (1596–1650)

Descartes made many contributions to philosophy, science, and mathematics. The idea of representing points in the plane by pairs of real numbers and representing curves in the plane by equations was described by Descartes in his book *La Géométrie*, published in 1637.

The Graph of an Equation

In 1637 the French mathematician René Descartes revolutionized the study of mathematics by joining its two major fields—algebra and geometry. With Descartes's coordinate plane, geometric concepts could be formulated analytically and algebraic concepts could be viewed graphically. Such is the power of this approach that within a century, much of calculus had been developed.

You can enhance your chance of success in calculus by following the same approach. That is, by viewing calculus from multiple perspectives—*graphically*, *analytically*, and *numerically*, you will increase your understanding of core concepts.

Consider the equation $3x + y = 7$. The point $(2, 1)$ is a **solution point** of the equation because the equation is satisfied (is true) when 2 is substituted for x and 1 is substituted for y. This equation has many other solutions, such as $(1, 4)$ and $(0, 7)$. To systematically find other solutions, solve the original equation for y.

$$y = 7 - 3x$$ Analytic approach

Then construct a **table of values** by substituting several values of x.

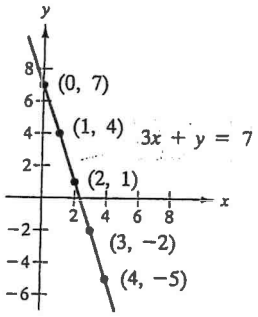
x	0	1	2	3	4
y	7	4	1	−2	−5

Numerical approach

From the table, you can see that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solutions of the original equation

$$3x + y = 7.$$ Original equation

Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph** of the equation, as shown in Figure P.1.



Graphical approach: $3x + y = 7$
Figure P.1

NOTE Even though we refer to the sketch shown in Figure P.1 as the graph of $3x + y = 7$, it really represents only a *portion* of the graph. The entire graph would extend beyond the page.

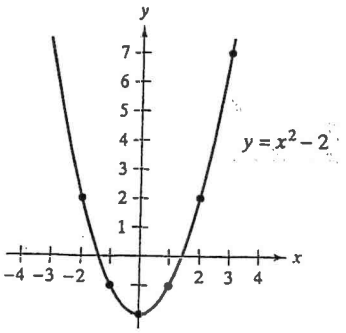
In this course, you will study many sketching aids. The simplest is point plotting—that is, you plot points until the basic shape of the graph seems apparent.

EXAMPLE 1 Sketching a Graph by Point Plotting

Sketch the graph of $y = x^2 - 2$.

Solution First construct a table of values. Then plot the points given in the table.

x	−2	−1	0	1	2	3
y	2	−1	−2	−1	2	7



The parabola $y = x^2 - 2$
Figure P.2

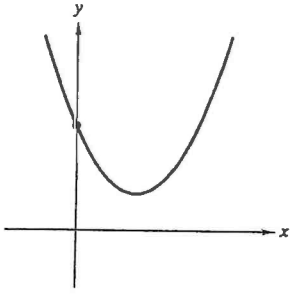
Finally, connect the points with a *smooth curve*, as shown in Figure P.2. This graph is a **parabola.** It is one of the conics you will study in Chapter 9.

Intercepts of a Graph

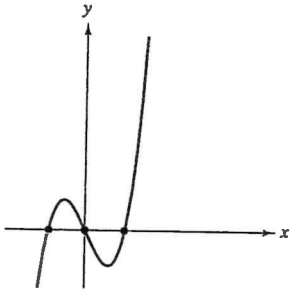
Two types of solution points that are especially useful are those having zero as their x- or y-coordinate. Such points are called **intercepts** because they are the points at which the graph intersects the x- or y-axis. The point $(a, 0)$ is an **x-intercept** of the graph of an equation if it is a solution point of the equation. To find the x-intercepts of a graph, let y be zero and solve the equation for x. The point $(0, b)$ is a **y-intercept** of the graph of an equation if it is a solution point of the equation. To find the y-intercepts of a graph, let x be zero and solve the equation for y.

NOTE Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, we will use the term *intercept* to mean either the point or the coordinate.

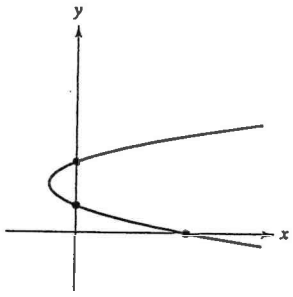
It is possible for a graph to have no intercepts, or it might have several. For instance, consider the four graphs shown in Figure P.5.



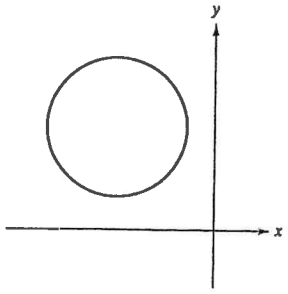
No x-intercepts
One y-intercept



Three x-intercepts
One y-intercept



One x-intercept
Two y-intercepts



No intercepts

Figure P.5

EXAMPLE 2 Finding x- and y-intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

Solution To find the x-intercepts, let y be zero and solve for x.

$$x^3 - 4x = 0 \qquad \text{Let } y \text{ be zero.}$$

$$x(x - 2)(x + 2) = 0 \qquad \text{Factor.}$$

$$x = 0, 2, \text{ or } -2 \qquad \text{Solve for } x.$$

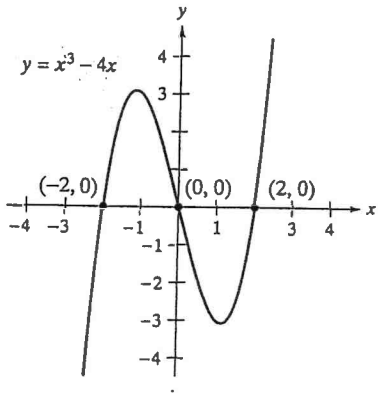
Because this equation has three solutions, you can conclude that the graph has three x-intercepts:

$$(0, 0), (2, 0), \text{ and } (-2, 0). \qquad x\text{-intercepts}$$

To find the y-intercepts, let x be zero. Doing this produces $y = 0$. Thus, the y-intercept is

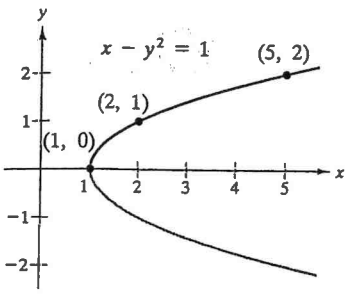
$$(0, 0). \qquad y\text{-intercept}$$

(See Figure P.6.)



ntercepts of a graph
igure P.6

TECHNOLOGY Example 2 uses an analytic approach to finding intercepts. When an analytic approach is not possible, you can use a graphical approach by finding the points where the graph intersects the axes. Try using a graphing utility to



First plot the points above the x-axis. Then use symmetry to complete the graph.
Figure P.9

EXAMPLE 4 Using Symmetry to Sketch a Graph

Sketch the graph of $x - y^2 = 1$.

Solution The graph is symmetric with respect to the x-axis because replacing y by $-y$ yields an equivalent equation.

$x - y^2 = 1$	Original equation
$x - (-y)^2 = 1$	Replace y by $-y$.
$x - y^2 = 1$	Equivalent equation

This means that the portion of the graph below the x-axis is a mirror image of the portion above the x-axis. To sketch the graph, first sketch the portion above the x-axis. Then reflect in the x-axis to obtain the entire graph, as shown in Figure P.9.

TECHNOLOGY Graphing utilities are designed so that they most easily graph equations in which y is a function of x (see Section P.3 for a definition of **function**). To graph other types of equations, you need to split the graph into two or more parts *or* you need to use a different graphing mode. For instance, to sketch the graph of the equation in Example 4, you can split it into two parts.

$y_1 = \sqrt{x - 1}$	Top portion of graph
$y_2 = -\sqrt{x - 1}$	Bottom portion of graph

Points of Intersection

A **point of intersection** of the graphs of two equations is a point that satisfies both equations. You can find the points of intersection of two graphs by solving their equations simultaneously.

EXAMPLE 5 Finding Points of Intersection

Find all points of intersection of the graphs of $x^2 - y = 3$ and $x - y = 1$.

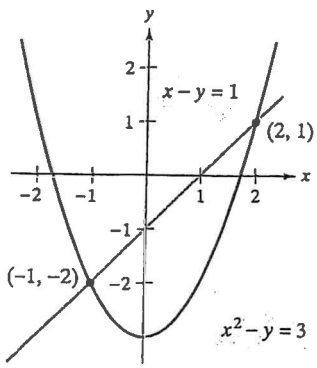
Solution It is helpful to begin by sketching the graphs of both equations on the *same* rectangular coordinate system, as shown in Figure P.10. Having done this, it appears that the two graphs have two points of intersection. To find these two points, you can use the following steps.

$y = x^2 - 3$	Solve first equation for y.
$y = x - 1$	Solve second equation for y.
$x^2 - 3 = x - 1$	Equate y-values.
$x^2 - x - 2 = 0$	Write in general form.
$(x - 2)(x + 1) = 0$	Factor.
$x = 2$ or -1	Solve for x.

The corresponding values of y are obtained by substituting $x = 2$ and $x = -1$ into either of the original equations. Doing this produces two points of intersection:

$(2, 1)$ and $(-1, -2)$. Points of intersection

You can check these points by substituting into *both* of the original equations or by using the trace feature of a graphing utility.

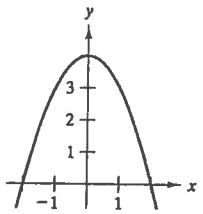


Two points of intersection
Figure P.10

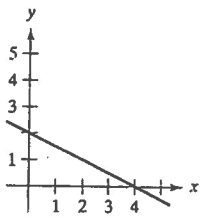
EXERCISES FOR SECTION P.1

In Exercises 1–4, match the equation with its graph. [Graphs are labeled (a), (b), (c), and (d).]

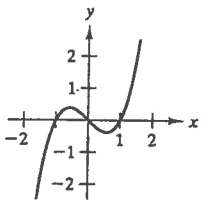
(a)



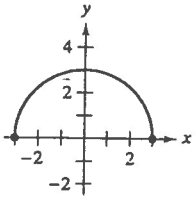
(b)



(c)



(d)



1. $y = -\frac{1}{2}x + 2$
2. $y = \sqrt{9 - x^2}$
3. $y = 4 - x^2$
4. $y = x^3 - x$

In Exercises 5–10, find any intercepts.

5. $y = x^2 + x - 2$
6. $y^2 = x^3 - 4x$
7. $y = x^2\sqrt{9 - x^2}$
8. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$
9. $x^2y - x^2 + 4y = 0$
10. $y = 2x - \sqrt{x^2 + 1}$

In Exercises 11–18, check for symmetry with respect to each axis and to the origin.

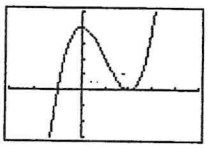
11. $y = x^2 - 2$
12. $xy - \sqrt{4 - x^2} = 0$
13. $y^2 = x^3 - 4x$
14. $xy^2 = -10$
15. $y = x^3 + x$
16. $xy = 1$
17. $y = \dfrac{x}{x^2 + 1}$
18. $y = x^3 + x - 3$

In Exercises 19–30, sketch the graph of the equation. Identify any intercepts and test for symmetry.

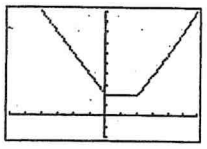
19. $y = -3x + 2$
20. $y = -\frac{1}{2}x + 2$
21. $y = \frac{1}{2}x - 4$
22. $y = x^2 + 3$
23. $y = 1 - x^2$
24. $y = 2x^2 + x$
25. $y = x^3 + 2$
26. $y = \sqrt{9 - x^2}$
27. $y = (x + 2)^2$
28. $x = y^2 - 4$
29. $y = \dfrac{1}{x}$
30. $y = 2x^4$

In Exercises 31 and 32, describe the viewing rectangle of a graphing utility that yields the figure.

31. $y = x^3 - 3x^2 + 4$



32. $y = |x| + |x - 10|$



In Exercises 33–36, use a graphing utility to graph the equation. (See Technology note, page 7.) Identify any intercepts and test for symmetry.

33. $y = -2x^2 + x + 1$
34. $y = x\sqrt{25 - x^2}$
35. $y = \dfrac{5}{x^2 + 1} - 1$
36. $x^2 + 4y^2 = 4$

In Exercises 37 and 38, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each given solution point accurate to two decimal places. (*Hint:* You may need to use the zoom feature to obtain the required accuracy.)

37. $y = \sqrt{5 - x}$ (a) $(2, y)$ (b) $(x, 3)$
38. $y = x^5 - 5x$ (a) $(-0.5, y)$ (b) $(x, -4)$

In Exercises 39–42, write an equation whose graph has the indicated property. (There may be more than one correct answer.)

39. The graph has intercepts at $x = -2$, $x = 4$, and $x = 6$.
40. The graph has intercepts at $x = -\frac{5}{2}$, $x = 2$, and $x = \frac{3}{2}$.
41. The graph is symmetric with respect to the origin.
42. The graph is symmetric with respect to the x-axis.

In Exercises 43–50, find the points of intersection of the graphs of the equations, and check your results analytically.

43. $x + y = 2$
    $2x - y = 1$
44. $2x - 3y = 13$
    $5x + 3y = 1$
45. $x + y = 7$
    $3x - 2y = 11$
46. $x^2 + y^2 = 25$
    $2x + y = 10$
47. $x^2 + y^2 = 5$
    $x - y = 1$
48. $x^2 + y = 4$
    $2x - y = 1$
49. $y = x^3$
    $y = x$
50. $x = 3 - y^2$
    $y = x - 1$

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

SECTION P.2 Linear Models and Rates of Change

The Slope of a Line • Equations of Lines • Ratios and Rates of Change • Graphing Linear Models • Parallel and Perpendicular Lines

The Slope of a Line

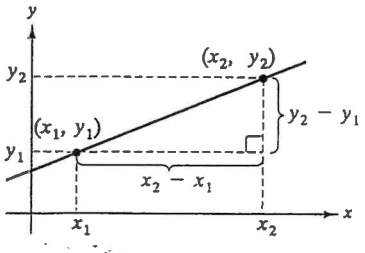
The **slope** of a nonvertical line is a measure of the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right. Consider the two points (x_1, y_1) and (x_2, y_2) on the line in Figure P.12. As you move from left to right along this line, a vertical change of

$$\Delta y = y_2 - y_1 \qquad \text{Change in } y$$

units corresponds to a horizontal change of

$$\Delta x = x_2 - x_1 \qquad \text{Change in } x$$

units. (Δ is a Greek uppercase letter *delta*, and the symbols Δy and Δx are read "delta y" and "delta x.")



$\Delta y = y_2 - y_1$ = change in y
$\Delta x = x_2 - x_1$ = change in x

Figure P.12

Definition of the Slope of a Line

The **slope** m of a nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2.$$

SYMBOL FOR SLOPE

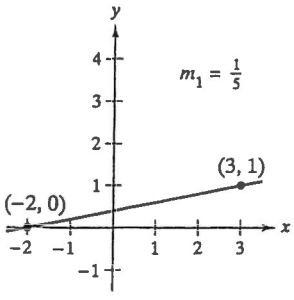
The use of the letter m to represent the slope of a line comes from *monter*, the French verb meaning to mount, to climb, or to rise.

NOTE When using the formula for slope, note that

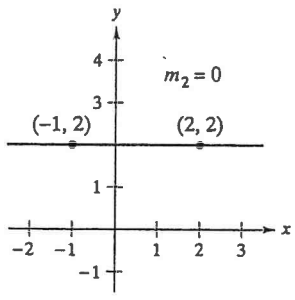
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{-(x_1 - x_2)} = \frac{y_1 - y_2}{x_1 - x_2}.$$

Thus, it does not matter in which order you subtract *as long as* you are consistent and both "subtracted coordinates" come from the same point.

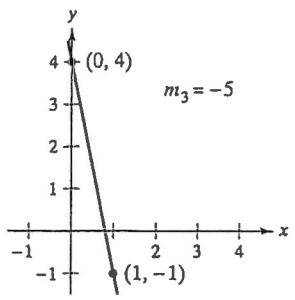
Figure P.13 shows four lines: one has a positive slope, one has a slope of zero, one has a negative slope, and one has an "undefined slope." In general, the greater the absolute value of the slope of a line, the steeper the line is. For instance, in Figure P.13, the line with a slope of -5 is steeper than the line with a slope of $\frac{1}{5}$.



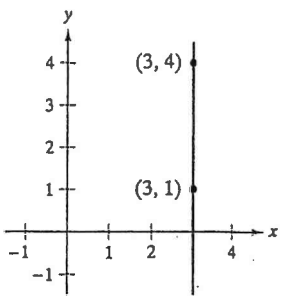
If m is positive, then the line rises from left to right.



If m is zero, then the line is horizontal.



If m is negative, then the line falls from left to right.



If m is undefined, then the line is vertical.

Figure P.13

Ratios and Rates of Change

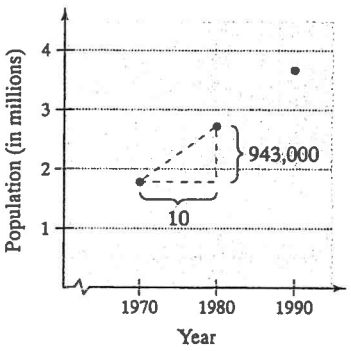
The slope of a line can be interpreted as either a *ratio* or a *rate*. If the x- and y-axes have the same unit of measure, the slope has no units and is a **ratio.** If the x- and y-axes have different units of measure, the slope is a rate or **rate of change.** In your study of calculus, you will encounter applications involving both interpretations of slope.

EXAMPLE 2 Population Growth and Engineering Design

a. The population of Arizona was 1,775,000 in 1970 and 2,718,000 in 1980. Over this 10-year period, the average rate of change of the population was

$$\text{Rate of change} = \frac{\text{change in population}}{\text{change in years}} = \frac{2{,}718{,}000 - 1{,}775{,}000}{1980 - 1970} = 94{,}300 \text{ people per year.}$$

If Arizona's population had continued to increase at this same rate for the next 10 years, it would have had a 1990 population of 3,661,000. In the 1990 census, however, Arizona's population was determined to be 3,665,000, so the population's rate of change from 1980 to 1990 was a little greater than in the previous decade (see Figure P.16).



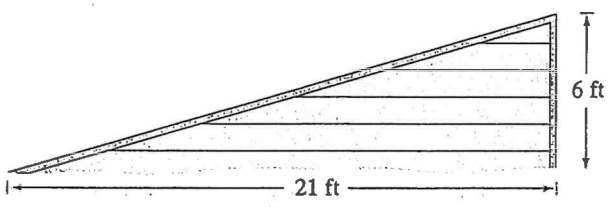
Population of Arizona in census years
Figure P.16

b. In tournament water-ski jumping, the ramp rises to a height of 6 feet on a raft that is 21 feet long, as shown in Figure P.17. The slope of the ski ramp is the ratio of its height (the rise) to the length of its base (the run).

$$\text{Slope of ramp} = \frac{\text{rise}}{\text{run}} = \frac{6 \text{ feet}}{21 \text{ feet}} = \frac{2}{7}$$

Rise is vertical change, run is horizontal change.

In this case, note that the slope is a ratio and has no units.



Dimensions of a water-ski ramp
Figure P.17

NOTE The rate of change found in Example 2a is an **average rate of change.** An average rate of change is always calculated over an interval. In this case, the interval is [1970, 1980]. In Chapter 2 you will study another type of rate of change called an *instantaneous rate of change.*

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of *any* line can be written in the **general form**

$$Ax + By + C = 0$$ General form of the equation of a line

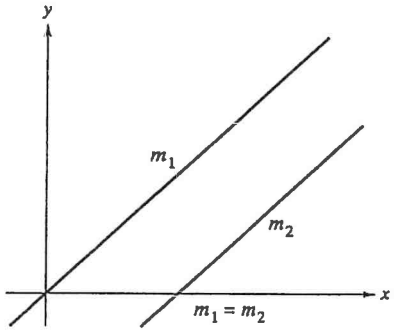
where A and B are not *both* zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

Summary of Equations of Lines

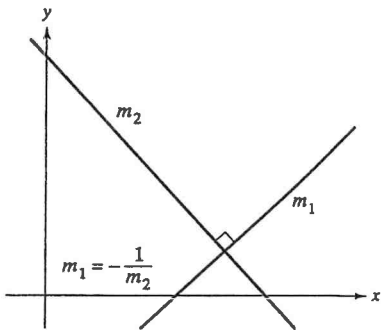
1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Point-slope form: $y - y_1 = m(x - x_1)$
5. Slope-intercept form: $y = mx + b$

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure P.19. Specifically, nonvertical lines with the same slope are parallel and nonvertical lines whose slopes are negative reciprocals are perpendicular.



Parallel lines



Perpendicular lines

Figure P.19

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

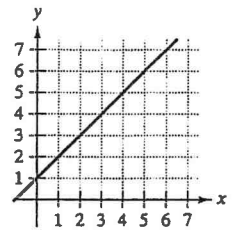
STUDY TIP In mathematics, the phrase "if and only if" is a way of stating two implications in one statement. For instance, the first statement at the right could be rewritten as the following two implications.

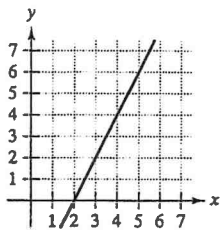
(a) If two distinct nonvertical lines are parallel, then their slopes are equal.

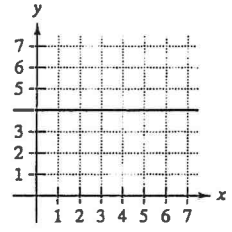
(b) If two distinct nonvertical lines have equal slopes, then they are parallel.

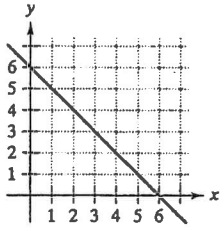
EXERCISES FOR SECTION P.2

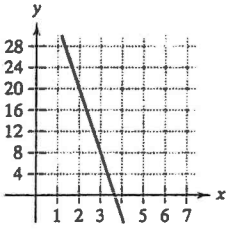
In Exercises 1–6, estimate the slope of the line from its graph.

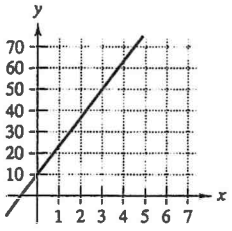
1. 

2. 

3. 

4. 

5. 

6. 

In Exercises 7 and 8, sketch the graph of the lines through the point with the indicated slopes. Make the sketches on the same set of coordinate axes.

	Point	*Slopes*			
7.	(2, 3)	(a) 0	(b) 1	(c) -2	(d) Undefined
8.	$(-4, 1)$	(a) 3	(b) -3	(c) $\frac{1}{3}$	(d) 0

In Exercises 9–12, plot the pair of points and find the slope of the line passing through them.

9. $(3, -4), (5, 2)$
10. $(2, 1), (2, 5)$
11. $(1, 2), (-2, 4)$
12. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 13–16, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)

	Point	*Slope*		*Point*	*Slope*
13.	(2, 1)	$m = 0$	14.	$(-3, 4)$	m undefined
15.	(1, 7)	$m = -3$	16.	$(-2, -2)$	$m = 2$

17. ***Modeling Data*** The table gives the earnings per share of common stock for General Mills for the years 1987 through 1994. Time in years is represented by t, with $t = 0$ corresponding to 1990, and the earnings per share are represented by y. *(Source: General Mills 1994 Annual Report)*

t	-3	-2	-1	0
y	\$1.25	\$1.63	\$2.53	\$2.32

t	1	2	3	4
y	\$2.87	\$2.99	\$3.10	\$2.95

(a) Plot the data by hand and connect adjacent points with a line segment.

(b) Use the slope to determine the years when earnings decreased most rapidly and increased most rapidly.

18. ***Conveyor Design*** A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.

(a) Find the slope of the conveyor.

(b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

19. ***Writing*** Write a short paragraph explaining whether or not any two points on a line can be used to calculate the slope of the line.

20. ***Rate of Change*** Each of the following is the slope of a line representing daily revenue y in terms of time x in days. Use the slope to interpret any change in daily revenue for a 1-day increase in time.

(a) $m = 400$ (b) $m = 100$ (c) $m = 0$

In Exercises 21–24, find the slope and the y-intercept (if possible) of the line.

21. $x + 5y = 20$
22. $6x - 5y = 15$
23. $x = 4$
24. $y = -1$

In Exercises 25–30, find an equation of the line that passes through the points, and sketch the line.

25. $(2, 1), (0, -3)$
26. $(-3, -4), (1, 4)$
27. $(0, 0), (-1, 3)$
28. $(-3, 6), (1, 2)$
29. $(1, -2), (3, -2)$
30. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 31–36, find an equation of the line that passes through the point and has the indicated slope. Sketch the line.

	Point	*Slope*		*Point*	*Slope*
31.	(0, 3)	$m = \frac{3}{4}$	32.	$(-1, 2)$	m undefined
33.	(0, 0)	$m = \frac{2}{3}$	34.	$(-2, 4)$	$m = -\frac{3}{5}$

69. ***Temperature Conversion*** Find the linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F) to convert 72°F to degrees Celsius.

70. ***Reimbursed Expenses*** A company reimburses its sales representatives \$150 per day for lodging and meals plus 30¢ per mile driven. Write a linear equation giving the daily cost C to the company in terms of x, the number of miles driven.

71. ***Career Choice*** An employee has two options for positions in a large corporation. One position pays \$12.50 per hour *plus* an additional unit rate of \$0.75 per unit produced. The other pays \$9.20 per hour *plus* a unit rate of \$1.30.

(a) Find linear equations for the hourly wages W in terms of x, the number of units produced per hour, for each of the options.

(b) Use a graphing utility to graph the linear equations and find the point of intersection.

(c) Interpret the meaning of the point of intersection of the graphs in part (b). How would you use this information to select the correct option if the goal were to obtain the highest hourly wage?

72. ***Straight-Line Depreciation*** A small business purchases a piece of equipment for \$875. After 5 years the equipment will be outdated, having no value.

(a) Write a linear equation giving the value y of the equipment in terms of the time x, $0 \le x \le 5$.

(b) Use a graphing utility to graph the equation.

(c) Use the trace feature to estimate (to two-decimal-place accuracy) the value of the equipment when $x = 2$.

(d) Use the trace feature to estimate (to two-decimal-place accuracy) the time when the value of the equipment is \$200.

73. ***Apartment Rental*** A real estate office handles an apartment complex with 50 units. When the rent is \$580 per month, all 50 units are occupied. However, when the rent is \$625, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear. (*Note:* Here we use the term *demand* to refer to the number of occupied units.)

(a) Write a linear equation giving the demand x in terms of the rent p.

(b) *Linear extrapolation* Use a graphing utility to graph the demand equation and use the trace feature to predict the number of units occupied if the rent is raised to \$655.

(c) *Linear interpolation* Predict the number of units occupied if the rent is lowered to \$595. Verify graphically.

74. ***Modeling Data*** An instructor gives regular 20-point quizzes and 100-point exams in a mathematics course. Average scores for six students, given as ordered pairs (x, y) where x is the average quiz score and y is the average test score, are (18, 87), (10, 55), (19, 96), (16, 79), (13, 76), and (15, 82).

(a) Use the regression capabilities of a graphing utility to find

(b) Use a graphing utility to plot the points and graph the regression line in the same viewing rectangle.

(c) Use the regression line to predict the average exam score for a student with an average quiz score of 17.

(d) Interpret the meaning of the slope of the regression line.

(e) If the instructor added 4 points to the average test score of everyone in the class, describe the change in the position of the plotted points and the change in the equation of the line.

Distance **In Exercises 75–80, find the distance between the point and line, or between the lines, using the formula for the distance between the point (x_1, y_1) and the line $Ax + By + C = 0$.**

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

75. Point: (0, 0)
Line: $4x + 3y = 10$

76. Point: (2, 3)
Line: $4x + 3y = 10$

77. Point: $(-2, 1)$
Line: $x - y - 2 = 0$

78. Point: (6, 2)
Line: $x = -1$

79. Line: $x + y = 1$
Line: $x + y = 5$

80. Line: $3x - 4y = 1$
Line: $3x - 4y = 10$

81. Show that the distance between the point (x_1, y_1) and the line $Ax + By + C = 0$ is

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

82. Write the distance d between the point (3, 1) and the line $y = mx + 4$ in terms of m. Use a graphing utility to graph the equation. When is the distance 0? Explain the result geometrically.

83. Prove that the diagonals of a rhombus intersect at right angles.

84. Prove that the figure formed by connecting consecutive midpoints of the sides of any quadrilateral is a parallelogram.

85. Prove that if the points (x_1, y_1) and (x_2, y_2) lie on the same line as (x_1^*, y_1^*) and (x_2^*, y_2^*), then

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Assume $x_1 \neq x_2$ and $x_1^* \neq x_2^*$.

86. Prove that if the slopes of two nonvertical lines are negative reciprocals of each other, then the lines are perpendicular.

True or False? **In Exercises 87 and 88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

87. The lines represented by $ax + by = c_1$ and $bx - ay = c_2$ are perpendicular. Assume $a \neq 0$ and $b \neq 0$.

88. It is possible for two lines with positive slopes to be perpendicular to each other.

In an equation that defines a function, the role of the variable x is simply that of a placeholder. For instance, the function given by

$$f(x) = 2x^2 - 4x + 1$$

can be described by the form

$$f(\quad) = 2(\quad)^2 - 4(\quad) + 1$$

where parentheses are used instead of x. To evaluate $f(-2)$, simply place -2 in each set of parentheses.

$$\begin{aligned} f(-2) &= 2(-2)^2 - 4(-2) + 1 && \text{Substitute } -2 \text{ for } x. \\ &= 2(4) + 8 + 1 && \text{Simplify.} \\ &= 17 && \text{Simplify.} \end{aligned}$$

NOTE Although f is often used as a convenient function name and x as the independent variable, you can use other symbols. For instance, the following equations all define the same function.

$f(x) = x^2 - 4x + 7$ Function name is f, independent variable is x.

$f(t) = t^2 - 4t + 7$ Function name is f, independent variable is t.

$g(s) = s^2 - 4s + 7$ Function name is g, independent variable is s.

EXAMPLE 1 Evaluating a Function

For the function f defined by $f(x) = x^2 + 7$, evaluate each of the following.

a. $f(3a)$ **b.** $f(b - 1)$ **c.** $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}, \quad \Delta x \neq 0$

Solution

a. $f(3a) = (3a)^2 + 7$ Substitute $3a$ for x.

$= 9a^2 + 7$ Simplify.

b. $f(b - 1) = (b - 1)^2 + 7$ Substitute $b - 1$ for x.

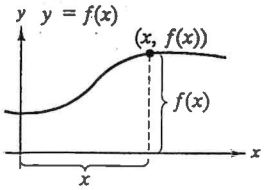
$= b^2 - 2b + 1 + 7$ Expand binomial.

$= b^2 - 2b + 8$ Simplify.

c. $$\begin{aligned} \frac{f(x + \Delta x) - f(x)}{\Delta x} &= \frac{[(x + \Delta x)^2 + 7] - (x^2 + 7)}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 7 - x^2 - 7}{\Delta x} \\ &= \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= 2x + \Delta x, \qquad \Delta x \neq 0 \end{aligned}$$

NOTE The expression in Example 1(c) is called a *difference quotient* and has a special significance in calculus. We will say more about this in Chapter 2.

The Graph of a Function



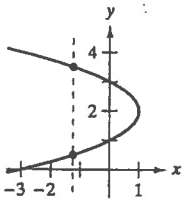
The graph of a function
Figure P.25

The graph of the function $y = f(x)$ consists of all points $(x, f(x))$, where x is in the domain of f. In Figure P.25, note that

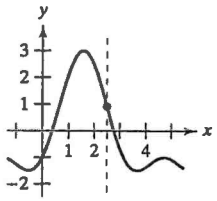
x = the directed distance from the y-axis

$f(x)$ = the directed distance from the x-axis.

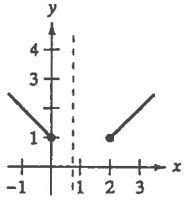
A vertical line can intersect the graph of a function of x at most *once*. This observation provides a convenient visual test (called the **vertical line test**) for functions of x. For example, in Figure P.26(a), you can see that the graph does not define y as a function of x because a vertical line intersects the graph twice, whereas in Figures P.26(b) and (c), the graphs do define y as a function of x.



(a) Not a function of x



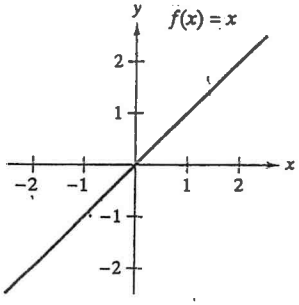
(b) A function of x



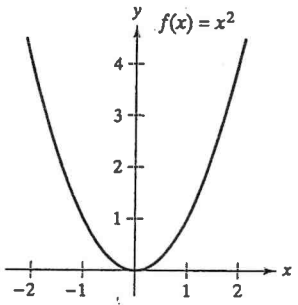
(c) A function of x

Figure P.26

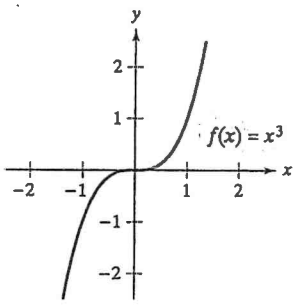
Figure P.27 shows the graphs of eight basic functions. You should be able to recognize these graphs. (Graphs of the other four basic trigonometric functions are shown in the appendix.)



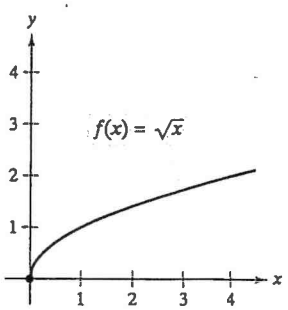
Identity function



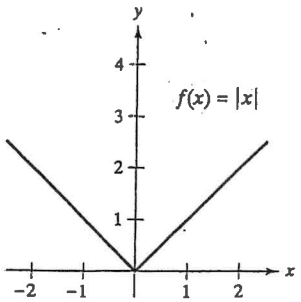
Squaring function



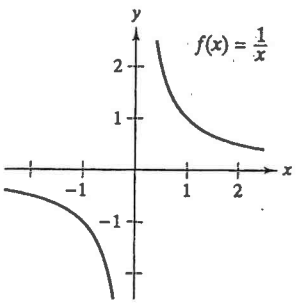
Cubing function



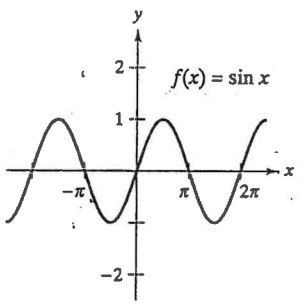
Square root function



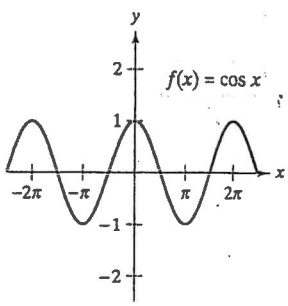
Absolute value function



Rational function



Sine function



Cosine function

The graphs of eight basic functions
Figure P.27



Leonhard Euler (1707–1783)

In addition to making major contributions to almost every branch of mathematics, Euler was one of the first to apply calculus to real-life problems in physics. His extensive published writings include such topics as shipbuilding, acoustics, optics, astronomy, mechanics, and magnetism.

Classifications and Combinations of Functions

The modern notion of a function is derived from the efforts of many seventeenth- and eighteenth-century mathematicians. Of particular note was Leonhard Euler, to whom we are indebted for the function notation $y = f(x)$. By the end of the eighteenth century, mathematicians and scientists had concluded that most real-world phenomena could be represented by mathematical models taken from a collection of functions called **elementary functions.** Elementary functions fall into three categories.

1. Algebraic functions (polynomial, radical, rational)
2. Trigonometric functions (sine, cosine, tangent, and so on)
3. Exponential and logarithmic functions

You can review the trigonometric functions in the appendix. The other nonalgebraic functions, such as the inverse trigonometric functions and the exponential and logarithmic functions, are introduced in Chapter 5.

The most common type of algebraic function is a **polynomial function**

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \qquad a_n \neq 0$$

where the positive integer n is the **degree** of the polynomial function. The numbers a_i are **coefficients,** with a_n the **leading coefficient** and a_0 the **constant term** of the polynomial function. It is common practice to use subscript notation for coefficients of general polynomial functions, but for polynomial functions of low degree, the following simpler forms are often used.

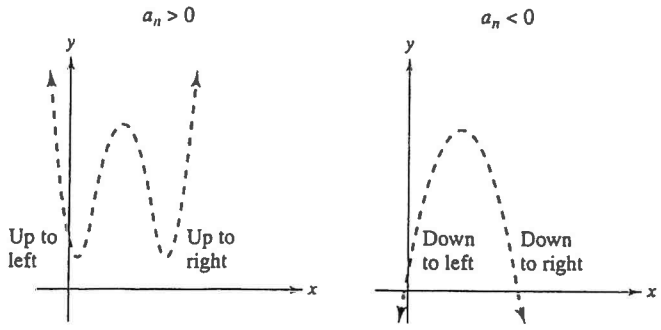
Zeroth degree:	$f(x) = a$	Constant function
First degree:	$f(x) = ax + b$	Linear function
Second degree:	$f(x) = ax^2 + bx + c$	Quadratic function
Third degree:	$f(x) = ax^3 + bx^2 + cx + d$	Cubic function

FOR FURTHER INFORMATION For more on the history of the concept of a function, see the article "Evolution of the Function Concept: A Brief Survey" by Israel Kleiner in the September 1989 issue of *The College Mathematics Journal.*

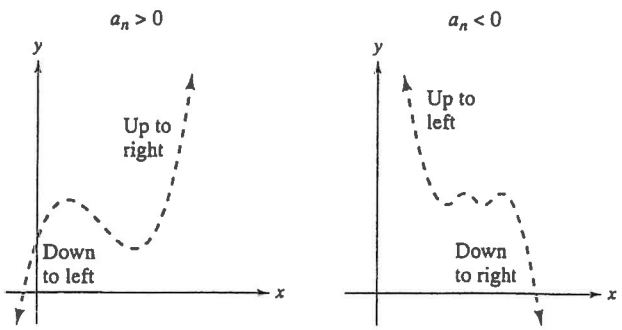
Although the graph of a polynomial function can have several turns, eventually the graph will rise or fall without bound as x moves to the right or left. Whether the graph of

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

eventually rises or falls can be determined by the function's degree (odd or even) and by the leading coefficient a_n, as indicated in Figure P.29. Note that the dashed portions of the graphs indicate that the **leading coefficient test** determines *only* the right and left behavior of the graph.



Graphs of polynomial functions of even degree



Graphs of polynomial functions of odd degree

The leading coefficient test for polynomial functions

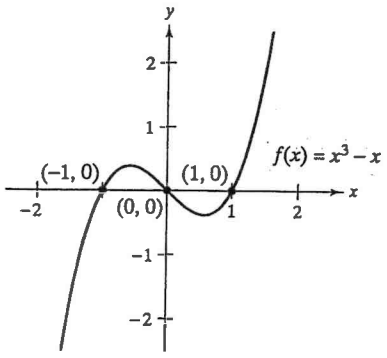
In Section P.1, an x-intercept of a graph was defined to be a point $(a, 0)$ at which the graph crosses the x-axis. If the graph represents a function f, the number a is a **zero** of f. In other words, *the zeros of a function f are the solutions of the equation $f(x) = 0$.* For example, the function $f(x) = x - 4$ has a zero at $x = 4$ because $f(4) = 0$.

In Section P.1 we also discussed different types of symmetry. In the terminology of functions, a function is **even** if its graph is symmetric with respect to the y-axis, and is **odd** if its graph is symmetric with respect to the origin. The symmetry tests in Section P.1 yield the following test for even and odd functions.

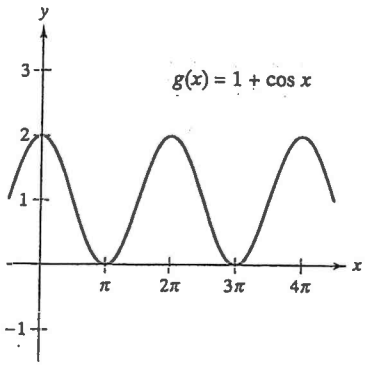
Test for Even and Odd Functions

The function $y = f(x)$ is **even** if $f(-x) = f(x)$.
The function $y = f(x)$ is **odd** if $f(-x) = -f(x)$.

NOTE Except for the constant function $f(x) = 0$, the graph of a function of x cannot have symmetry with respect to the x-axis because it then would fail the vertical line test for the graph of the function.



(a) Odd function



(b) Even function

Figure P.31

EXAMPLE 5 Even and Odd Functions and Zeros of Functions

Determine whether each function is even, odd, or neither. Then find the zeros of the function.

a. $f(x) = x^3 - x$ **b.** $g(x) = 1 + \cos x$

Solution

a. This function is odd because

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x).$$

The zeros of f are found as follows.

$$x^3 - x = 0 \qquad \text{Let } f(x) = 0.$$
$$x(x^2 - 1) = x(x - 1)(x + 1) = 0 \qquad \text{Factor.}$$
$$x = 0, 1, -1 \qquad \text{Zeros of } f$$

See Figure P.31(a).

b. This function is even because

$$g(-x) = 1 + \cos(-x) = 1 + \cos x = g(x). \qquad \cos(-x) = \cos(x)$$

The zeros of f are found as follows.

$$1 + \cos x = 0 \qquad \text{Let } g(x) = 0.$$
$$\cos x = -1 \qquad \text{Subtract 1 from each side.}$$
$$x = (2n + 1)\pi, \ n \text{ is an integer.} \qquad \text{Zeros of } g$$

See Figure P.31(b).

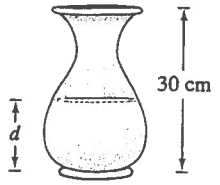
NOTE Each of the functions in Example 5 is either even or odd. However, some functions, such as $f(x) = x^2 + x + 1$, are neither even nor odd.

33. ***Think About It*** Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown in the figure to answer the questions if d is the depth of the water in centimeters and t is the time in seconds.

(a) Explain why d is a function of t.

(b) Determine the domain and range of the function.

(c) Sketch a possible graph of the function.



34. ***Think About It*** A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

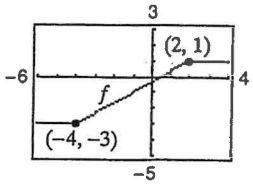
35. Select the viewing rectangle on a graphing utility that shows the most complete graph of the function $f(x) = 10x\sqrt{400 - x^2}$.

Xmin = -5	Xmin = -20	Xmin = -25
Xmax = 50	Xmax = 20	Xmax = 25
Xscl = 5	Xscl = 2	Xscl = 5
Ymin = -5000	Ymin = -500	Ymin = -2000
Ymax = 5000	Ymax = 500	Ymax = 2000
Yscl = 500	Yscl = 50	Yscl = 200

36. Use the graph of f shown in the figure to sketch the graph of each function.

(a) $f(x - 4)$ (b) $f(x + 2)$ (c) $f(x) + 4$

(d) $f(x) - 1$ (e) $2f(x)$ (f) $\frac{1}{2}f(x)$



37. Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. In each case, describe the transformation.

(a) $y = \sqrt{x} + 2$ (b) $y = -\sqrt{x}$ (c) $y = \sqrt{x - 2}$

38. Specify the sequence of transformations that will yield each graph of h from the graph of the function $f(x) = \sin x$.

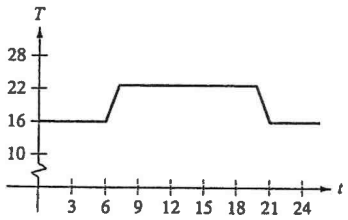
(a) $h(x) = \sin\left(x + \frac{\pi}{2}\right) + 1$ (b) $h(x) = -\sin(x - 1)$

39. ***Graphical Reasoning*** An electronically controlled thermostat in a home is programmed to automatically lower the temperature during the night (see figure). The temperature T in degrees Celsius is given in terms of t, the time in hours on a 24-hour clock.

(a) Approximate $T(4)$ and $T(15)$.

(b) Suppose the thermostat were reprogrammed to produce a temperature $H(t) = T(t - 1)$. How would this change the temperature in the house? Explain.

(c) Suppose the thermostat were reprogrammed to produce a temperature $H(t) = T(t) - 1$. How would this change the temperature in the house? Explain.



40. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the following.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$

(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

In Exercises 41–44, find the composite functions $(f \circ g)$ and $(g \circ f)$. What is the domain of each composite function? Are the two composite functions equal?

41. $f(x) = x^2$, $g(x) = \sqrt{x}$

42. $f(x) = x^2 - 1$, $g(x) = \cos x$

43. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

44. $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x + 2}$

45. ***Ripples*** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in feet) of the outer ripple is given by $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.

46. ***Automobile Aerodynamics*** The number of horsepower H required to overcome wind drag on a certain automobile is approximated by

$$H(x) = 0.002x^2 + 0.005x - 0.029, \quad 10 \le x \le 100$$

where x is the speed of the car in miles per hour.

(a) Use a graphing utility to graph H.

(b) Rewrite the power function so that x represents the speed in kilometers per hour. [Find $H(1.6x)$.]

In Exercises 47–50, determine whether the function is even, odd, or neither. Use a graphing utility to verify your result.

47. $f(x) = 4 - x^2$

48. $f(x) = \sqrt[3]{x}$

49. $f(x) = x \cos x$

50. $f(x) = \sin^2 x$