

UNITY CHRISTIAN SCHOOL
SUMMER REVIEW PACKET

For students in entering College Algebra

Name: _____

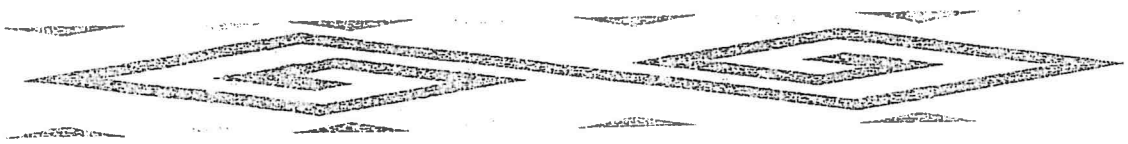
Are You Ready for College Algebra?

Pre-Calculus is different than previous math classes. The class will cover material faster and at the same time delve more deeply into the material and pay more attention to detail than in previous math classes. We will also work to make connections.

This packet contains problems, formulas, concepts, and examples that cover prerequisite skills necessary before entering Pre-Calculus. Please study and work through these problems carefully. By doing so, you will put yourself at an advantage, as the first month of Pre-Calculus can sometimes be the most difficult. You are to complete this packet over the summer and be ready to turn in your work <u>on the first day of school</u>. This will be graded for correctness. You will have a quiz over this material the first week of school. I will not be re-teaching this material—this is material that should have been mastered before Pre-Calculus and is necessary for a successful year.

Blessings and have a great summer!

1. This packet is to be handed in to your Pre-Calculus teacher on the first day of the school year.
2. All work must be shown in the packet.
3. This will be graded for correctness.



Radicals:

To simplify means that 1) no radicand has a perfect square factor and
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the *conjugate* of the denominator
The *conjugate* of $3 + \sqrt{2}$ is $3 - \sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

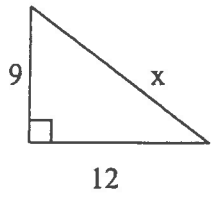
8a. $\frac{1}{\sqrt{2}}$
b. $\frac{2}{\sqrt{3}}$
c. $\frac{3}{2 - \sqrt{5}}$

Geometry:

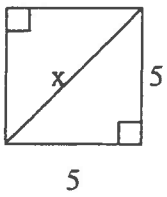
Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

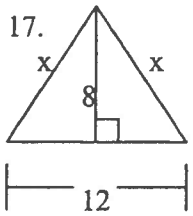
15.



16.

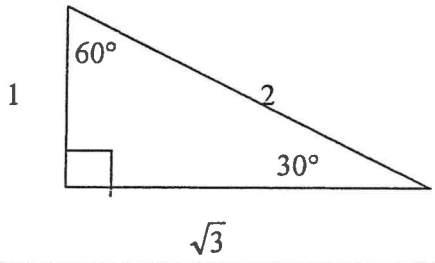


17.

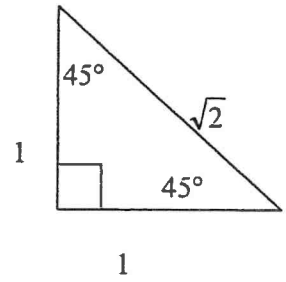


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.

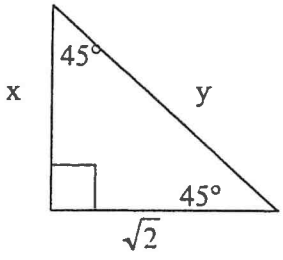


*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.

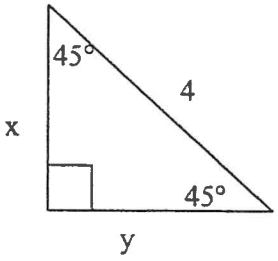


Solve for x and y.

19.



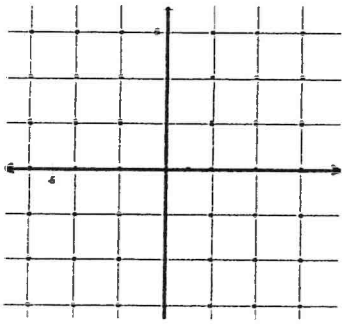
20.



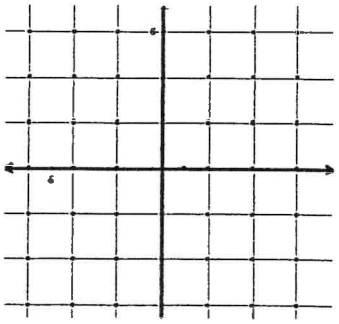
Graphing:

Graph each function, inequality, and / or system.

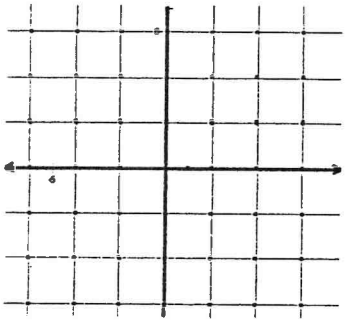
29. $3x-4y=12$



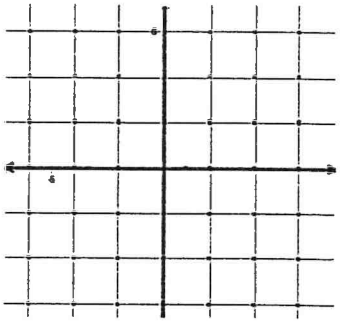
30. $\begin{cases} 2x+y=4 \\ x-y=2 \end{cases}$



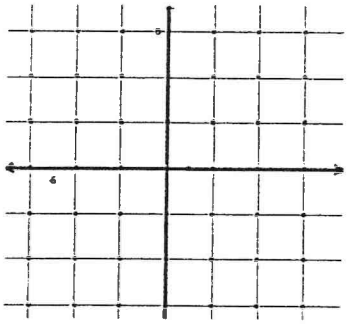
31. $y<-4x-2$



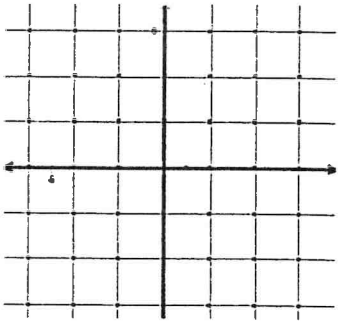
32. $y+2=|x+1|$



33. $y>|x|-1$



34. $y+4=(x-1)^2$



Vertex: _____

x-intercept(s): _____

y-intercept(s): _____

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\frac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a.) If you have one (other than 1) factor it out front.
b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EX: $m^3 + 64 = (m+4)(m^2-4m+16)$

$p^3 - 125 = (p-5)(p^2+5p+25)$

3 Terms

$x^2 + bx + c = (x+\ \ )(x+\ \ )$ **Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx + c = (x-\ \ )(x-\ \ )$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x-\ \ )(x+\ \ )$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x-\ \ )(x+\ \ )$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

a.) Pair up first two terms and last two terms
b.) Factor out GCF of each pair of numbers.
c.) Factor out front the parentheses that the terms have in common.
d.) Put leftover terms in parentheses.

Ex: $x^3 + 3x^2 + 9x + 27 = \left(x^3 + 3x^2\right) + \left(9x + 27\right)$

$= x^2\left(x+3\right) + 9\left(x+3\right)$

$= \left(x+3\right)\left(x^2+9\right)$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

QUADRATIC FORMULA – allows you to solve any quadratic for all its real and imaginary roots. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation: $x^2 + 2x + 3 = 0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2 + 2x + 3 = 0$ Determine values for a, b, and c.

$a = 1 \quad b = 2 \quad c = 3$ Find dicriminant.

$D = 2^2 - 4 \cdot 1 \cdot 3$

$D = 4 - 12$

$D = -8$ *There are two imaginary roots.*

Solve: $x = \dfrac{-2 \pm \sqrt{-8}}{2}$

$x = \dfrac{-2 \pm 2i\sqrt{2}}{2}$

$x = -1 \pm i\sqrt{2}$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

64. $5x^2 - 2x + 4 = 0$

Discriminant = _____

Type of Roots: _____

Roots = _____

Discriminant = _____

Type of Roots: _____

Roots = _____

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Suppose $f(x) = 2x$, $g(x) = 3x - 2$, *and* $h(x) = x^2 - 4$ **. Find the following:**

70. $f[g(2)] =$ _____

71. $f[g(x)] =$ _____

72. $f[h(3)] =$ _____

73. $g[f(x)] =$ _____

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y + 1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$ **, if possible.**

74. $f(x) = 5x + 2$

75. $f(x) = \frac{1}{2}x - \frac{1}{3}$

Addition and Subtraction.
First, find the least common denominator.
Write each fraction with the LCD.
Add / subtract numerators as indicated and leave the denominators as they are.

EX: $\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$ *Factor denominator completely.*

$=\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}$ *Find LCD* $(2x)(x+2)$

$=\frac{2(3x+1)}{2x(x+2)}+\frac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD as the denominator.*

$=\frac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$=\frac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}+\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$ *Find LCD first.* $x(x+2)$

$x(x+2)\left(\frac{5}{x+2}\right)+x(x+2)\left(\frac{1}{x}\right)=\left(\frac{5}{x}\right)x(x+2)$ *Multiply each term by the LCD.*

$5x+1(x+2)=5(x+2)$ *Simplify and solve.*

$5x+x+2=5x+10$

$6x+2=5x+10$

EX: $x=8$ ⇐ *Check your answer. Sometimes they do not check!*

Check :

$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{8}=\frac{5}{8}$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$

Solve and graph the following. Express your solution using interval notation.

91. $4-7x \geq 11$

92. $0.4x+5 \leq 1.2x-4$

93. $5[3m-(m+4)] > -2(m-4)$

94. $2x+1 \geq 6x-9$