UNITY CHRISTIAN SCHOOL
SUMMER REVIEW PACKET

For students in entering ***PRE-CALCULUS***

Name: ____

Are You Ready for Pre-Calculus?

Pre-Calculus is different than previous math classes. The class will cover material faster and at the same time delve more deeply into the material and pay more attention to detail than in previous math classes. We will also work to make connections.

This packet contains problems, formulas, concepts, and examples that cover prerequisite skills necessary before entering Pre-Calculus. Please study and work through these problems carefully. By doing so, you will put yourself at an advantage, as the first month of Pre-Calculus can sometimes be the most difficult. You are to complete this packet over the summer and be ready to turn in your work on the first day of school. This will be graded for correctness. You will have a quiz over this material the first week of school. I will not be re-teaching this material—this is material that should have been mastered before Pre-Calculus and is necessary for a successful year.

Blessings and have a great summer!

1. This packet is to be handed in to your Pre-Calculus teacher on the first day of the school year.
2. All work must be shown in the packet.
3. This will be graded for correctness.

Radicals:

To simplify means that 1) no radicand has a perfect square factor <u>and</u>
2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

<u>Examples</u>: Simplify $\sqrt{24}=\sqrt{4}\cdot\sqrt{6}$ find a perfect square factor
$=2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}}=\frac{\sqrt{7}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both the numerator and the denominator by $\sqrt{2}$

$=\frac{\sqrt{14}}{\sqrt{4}}=\frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
multiply the numerator and the denominator by the *conjugate* of the denominator
The *conjugate* of $3+\sqrt{2}$ is $3-\sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60}\cdot\sqrt{105}$

7. $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Rationalize.

8a. $\frac{1}{\sqrt{2}}$

b. $\frac{2}{\sqrt{3}}$

c. $\frac{3}{2-\sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$
$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice
$= i^2\sqrt{25}$ Simplify
$= (-1)(5) = -5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute
$= 6i+2i^2$ Simplify
$= 6i+2(-1)$ Make substitution
$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

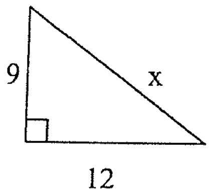
14. $\dfrac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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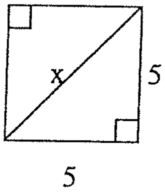

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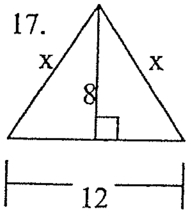
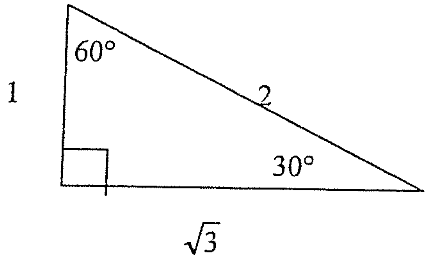
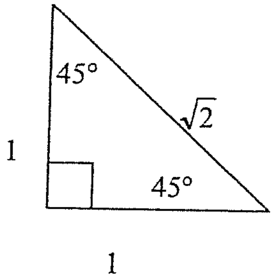


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles,
sides are in proportion $1, \sqrt{3}, 2$.

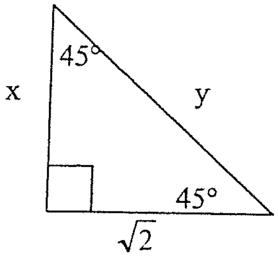


*In $45° - 45° - 90°$ triangles,
sides are in proportion $1, 1, \sqrt{2}$.



Solve for x and y.

19.



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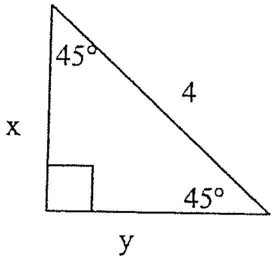

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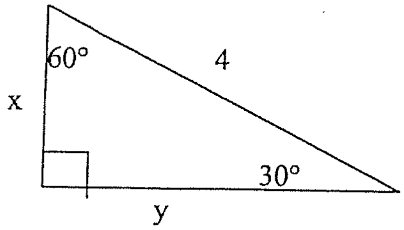

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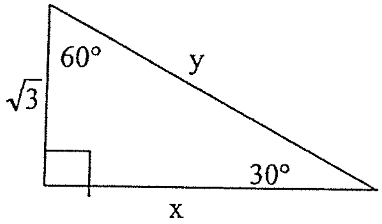


Equations of Lines:

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By = C$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$.

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

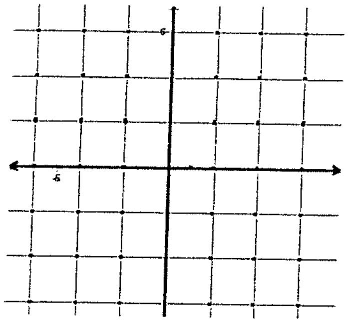
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

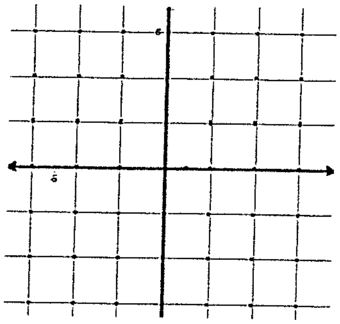
Graphing:

Graph each function, inequality, and / or system.

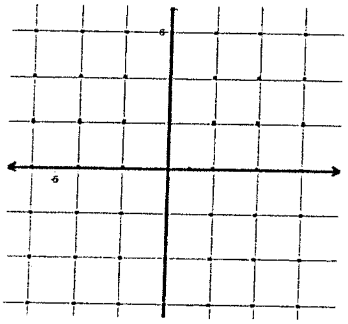
29. $3x-4y=12$



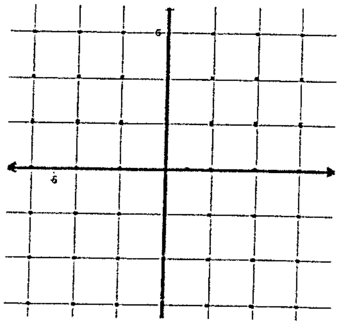
30. $\begin{cases}2x+y=4\\x-y=2\end{cases}$



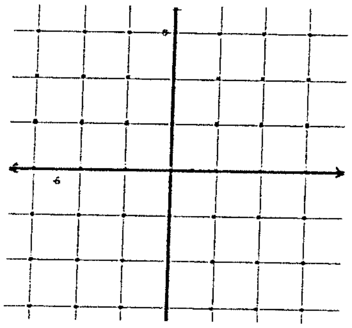
31. $y<-4x-2$



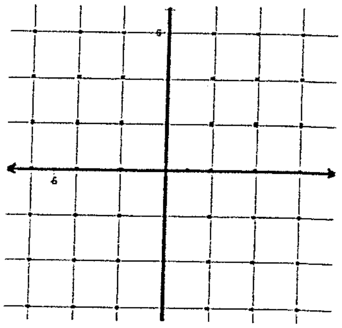
32. $y+2=|x+1|$



33. $y>|x|-1$



34. $y+4=(x-1)^2$



Vertex: ____________

x-intercept(s): ____________

y-intercept(s): ____________

Systems of Equations:

$3x + y = 6$

$2x - 2y = 4$

Substitution:
Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable.

Elimination:
Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$ solve 1st equation for y
$2x - 2(6 - 3x) = 4$ plug into 2nd equation
$2x - 12 + 6x = 4$ distribute
$8x = 16$ simplify
$x = 2$

$6x + 2y = 12$ multiply 1st equation by 2
$\underline{2x - 2y = 4}$ coefficients of y are opposite
$8x = 16$ add
$x = 2$ simplify

Plug $x = 2$ back into original

$$\begin{aligned} 3(2) + y &= 6 \\ 6 + y &= 6 \\ y &= 0 \end{aligned}$$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\frac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x - 3y + 6 - (6y + 4x - 9)$ *Distribute the negative through the parantheses.*

$= 8x - 3y + 6 - 6y - 4x + 9$ *Combine terms with similar variables.*

$= 8x - 4x - 3y - 6y + 6 + 9$

$= 4x - 9y + 15$

Simplify.

46. $3x^3 + 9 + 7x^2 - x^3$

47. $7m - 6 - (2m + 5)$

To multiplying two binomials, use FOIL.

EX: $(3x - 2)(x + 4)$ *Multiply the first, outer, inner, then last terms.*

$= 3x^2 + 12x - 2x - 8$ *Combine like terms.*

$= 3x^2 + 10x - 8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

- a.) If you have one (other than 1) factor it out front.
- b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

EX: $m^3 + 64 = (m+4)(m^2-4m+16)$
$p^3 - 125 = (p-5)(p^2+5p+25)$

3 Terms

$x^2 + bx + c = (x+\quad)(x+\quad)$ **Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx + c = (x-\quad)(x-\quad)$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x-\quad)(x+\quad)$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x-\quad)(x+\quad)$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

- a.) Pair up first two terms and last two terms
- b.) Factor out GCF of each pair of numbers.
- c.) Factor out front the parentheses that the terms have in common.
- d.) Put leftover terms in parentheses.

Ex:
$$\begin{aligned} x^3 + 3x^2 + 9x + 27 &= (x^3 + 3x^2) + (9x + 27) \\ &= x^2(x+3) + 9(x+3) \\ &= (x+3)(x^2+9) \end{aligned}$$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x+3)(x-7) = 0$ *Set each factor equal to zero.*

$x + 3 = 0 \quad x - 7 = 0$ *Solve each for x.*

$x = -3 \quad x = 7$

Solve each equation.

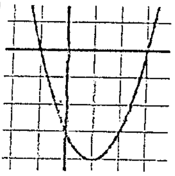
60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

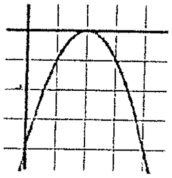
62. $x^2 - 14x + 40 = 0$

DISCRIMINANT: The number under the radical in the quadratic formula $(b^2 - 4ac)$ can tell you what kinds of roots you will have.

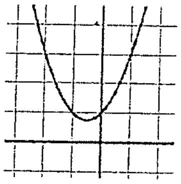
IF $b^2 - 4ac > 0$ you will have TWO real roots.
(touches x-axis twice)



IF $b^2 - 4ac = 0$ you will have ONE real root
(touches the x-axis once)



IF $b^2 - 4ac < 0$ you will have TWO imaginary roots.
(Graph does not cross the x-axis)



QUADRATIC FORMULA – allows you to solve any quadratic for all its real and imaginary roots. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

EX: In the equation: $x^2+2x+3=0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2+2x+3=0$ Determine values for a, b, and c.

$a=1 \quad b=2 \quad c=3$ Find dicriminant.

$D=2^2-4\cdot1\cdot3$

$D=4-12$

$D=-8$ *There are two imaginary roots.*

$$\text{Solve}: \quad x=\frac{-2\pm\sqrt{-8}}{2}$$

$$x=\frac{-2\pm 2i\sqrt{2}}{2}$$

$$x=-1\pm i\sqrt{2}$$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2-9x+14=0$

Discriminant = ______________

Type of Roots: ______________

Roots = ______________

64. $5x^2-2x+4=0$

Discriminant = ______________

Type of Roots: ______________

Roots = ______________

Long Division – can be used when dividing any polynomials.
Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$\begin{array}{rl} & 2x^2-3x+3+\dfrac{1}{x+3} \\ = x+3\,\big) & \overline{2x^3+3x^2-6x+10} \\ & \underline{(-)\ (2x^3+6x^2)} \\ & -3x^2-6x \\ & \underline{(-)\ (-3x^2-9x)} \\ & 3x+10 \\ & \underline{(-)\ (3x+9)} \\ & 1 \end{array}$$

Synthetic Division

$\dfrac{2x^3+3x^2-6x+10}{x+3}$

$$\begin{array}{r|rrrr} \underline{-3} & 2 & 3 & -6 & 10 \\ & \downarrow & -6 & 9 & -9 \\ \hline & 2 & -3 & 3 & 1 \end{array}$$

$= 2x-3x+3+\dfrac{1}{x+3}$

Divide each polynomial using long division OR synthetic division.

65. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

66. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for a given value, simply plug the value into the function for x.

Evaluate each function for the given value.

67. $f(x)=x^2-6x+2$

$f(3)=$ ________

68. $g(x)=6x-7$

$g(x+h)=$ ________

69. $f(x)=3x^2-4$

$5\left[f(x+2)\right]=$ ________

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Suppose $f(x)=2x,\ g(x)=3x-2,$ *and* $h(x)=x^2-4$ **. Find the following:**

70. $f[g(2)]=$ ________

71. $f[g(x)]=$ ________

72. $f[h(3)]=$ ________

73. $g[f(x)]=$ ________

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

74. $f(x)=5x+2$

75. $f(x)=\frac{1}{2}x-\frac{1}{3}$

Rational Algebraic Expressions:

Multiplying and Dividing.

Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2+10x+21}{5-4x-x^2} \bullet \frac{x^2+2x-15}{x^3+4x^2-21x}$$ *Factor everything completely.*

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \frac{(x+5)(x-3)}{x(x-3)(x+7)}$$ *Cancel out common factors in the top and bottom.*

$$=\frac{(x+3)}{x(1-x)}$$ *Simplify.*

Simplify.

76. $\frac{5z^3+z^2-z}{3z}$

77. $\frac{m^2-25}{m^2+5m}$

78. $\frac{10r^5}{21s^2} \bullet \frac{3s}{5r^3}$

79. $\frac{a^2-5a+6}{a+4} \bullet \frac{3a+12}{a-2}$

80. $\frac{6d-9}{5d+1} \div \frac{6-13d+6d^2}{15d^2-7d-2}$

Addition and Subtraction.

First, find the least common denominator.

Write each fraction with the LCD.

Add / subtract numerators as indicated and leave the denominators as they are.

EX: $\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$ *Factor denominator completely.*

$=\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}$ *Find LCD* $(2x)(x+2)$

$=\frac{2(3x+1)}{2x(x+2)}+\frac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD as the denominator.*

$=\frac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$=\frac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}+\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

Complex Fractions.

Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify as you did above

EX:

$$\frac{1+\frac{1}{a}}{\frac{2}{a^2}-1} \qquad \textit{Find LCD}: a^2$$

$$=\frac{\left(1+\frac{1}{a}\right)\cdot a^2}{\left(\frac{2}{a^2}-1\right)\cdot a^2} \qquad \textit{Multiply top and bottom by LCD.}$$

$$=\frac{a^2+a}{2-a^2} \qquad \textit{Factor and simplify if possible.}$$

$$=\frac{a(a+1)}{2-a^2}$$

84. $\dfrac{1-\frac{1}{2}}{2+\frac{1}{4}}$

85. $\dfrac{1+\frac{1}{z}}{z+1}$

86. $\dfrac{5+\frac{1}{m}-\frac{6}{m^2}}{\frac{2}{m}-\frac{2}{m^2}}$

87. $\dfrac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$ *Find LCD first.* $x(x+2)$

$x(x+2)\left(\frac{5}{x+2}\right)+x(x+2)\left(\frac{1}{x}\right)=\left(\frac{5}{x}\right)x(x+2)$ *Multiply each term by the LCD.*

$5x+1(x+2)=5(x+2)$ *Simplify and solve.*

$5x+x+2=5x+10$

$6x+2=5x+10$

EX: $x=8$ $\Leftarrow$ *Check your answer. Sometimes they do not check!*

Check:

$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$

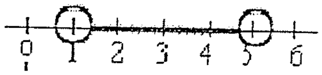
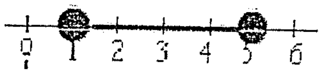
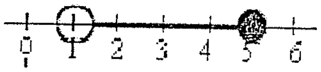
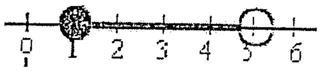
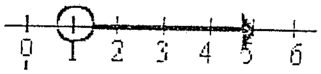
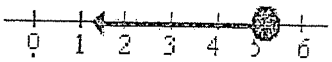
$\frac{5}{8}=\frac{5}{8}$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$

Interval Notation:	
Open Interval: (a, b) is interpreted as $a < x < b$ where the endpoints are NOT included. (While this notation resembles an ordered pair, in this context it refers to the interval upon which you are working.)	$(1, 5)$ 
Closed Interval: $[a, b]$ is interpreted as $a \leq x \leq b$ where the endpoints are included.	$[1, 5]$ 
Half-Open Interval: $(a, b]$ is interpreted as $a < x \leq b$ where a is not included, but b is included.	$(1, 5]$ 
Half-Open Interval: $[a, b)$ is interpreted as $a \leq x < b$ where a is included, but b is not included.	$[1, 5)$ 
Non-ending Interval: (a, ∞) is interpreted as $x > a$ where a is not included and infinity is always expressed as being "open" (not included).	$(1, \infty)$ 
Non-ending Interval: $(-\infty, b]$ is interpreted as $x \leq b$ where b is included and again, infinity is always expressed as being "open" (not included).	$(-\infty, 5]$ 

Inequalities:

To solve an inequality, simply follow the steps for solving an equation. The important difference is that multiplying or dividing both sides of the inequality by a negative number reverses the direction of the inequality symbol.

Example: Solve $6 + 5(2 - x) \leq 41$

$6 + 5(2 - x) \leq 41$

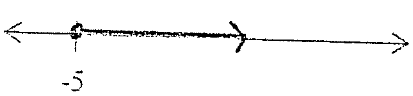
$6 + 10 - 5x \leq 41$

$16 - 5x \leq 41$

$-5x \leq 25$

$x \geq -5$

$[-5, \infty)$



Solve and graph the following. Express your solution using interval notation.

91. $4-7x \geq 11$

92. $0.4x+5 \leq 1.2x-4$

93. $5[3m-(m+4)] > -2(m-4)$

94. $2x+1 \geq 6x-9$